



Roll No. \_\_\_\_\_ to be filled in by the candidate

(For All Sessions)

Time: 30 Minutes

Marks: 20

**Mathematics (Objective)** *Lwp-12-1-23*

(Group-I)

Note: Write Answers to the Questions on the objective answer sheet provided. Four possible answers A, B, C and D to each question are given. Which answer you consider correct, fill the corresponding circle A, B, C or D given in front of each question with Marker or Pen ink on the answer sheet provided.

- 1.1  $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} = ?$  (A) 1 (B) 2 (C) 0 (D) -2
2.  $\cos hx + \sin hx = ?$  (A)  $e^x$  (B)  $e^{-x}$  (C)  $e^{2x}$  (D)  $2e^x$
3.  $\frac{d}{dx} [\ln(2^x)] = ?$  (A)  $\ln 2$  (B)  $2^x$  (C)  $\frac{1}{2^x}$  (D)  $\frac{\ln 2}{2^x}$
4.  $\frac{d}{dx} (\cos hx) = ?$  (A)  $-\sin hx$  (B)  $\sin hx$  (C)  $\operatorname{sech} x$  (D)  $\operatorname{cosech} x$
5. If  $f(x) = \sqrt{x}$ , then  $f^{-1}(0) = ?$  (A) 0 (B) 1 (C) Undefined (D)  $\frac{1}{2}$
6.  $\frac{d}{dx} (\sin^{-1} x + \cos^{-1} x) = ?$  (A) 1 (B) 0 (C) -1 (D) 2
7.  $\int dx = ?$  (A)  $x$  (B)  $\frac{1}{x}$  (C)  $x^2$  (D)  $\frac{1}{x^2}$
8.  $\int e^x(x+1) dx = ?$  (A)  $xe^x + c$  (B)  $e^x + c$  (C)  $x + c$  (D)  $x^2 + c$
9.  $\int_0^{\pi/2} \cos x dx = ?$  (A) 0 (B) -1 (C) 1 (D) 2
10.  $\int \frac{\sin 2x}{\sin x} dx = ?$  (A)  $2 \cos x + c$  (B)  $2 \sin x + c$  (C)  $\frac{1}{2} \sin x + c$  (D)  $\frac{1}{2} \cos x + c$
11. The slope of a line  $x = 5$  is: (A) 0 (B) 1 (C) -1 (D) Infinite
12. Midpoint of (0, -2) and (-2, 0) is: (A) (0, 0) (B) (-1, -1) (C) (-2, -2) (D) (0, -1)
13. Distance between (-1, 2) & (7, 5) is: (A)  $\sqrt{73}$  (B) 7 (C)  $2\sqrt{73}$  (D) 73
14. The solution of inequality  $x + 2y < 6$  is: (A) (1, 4) (B) (1, 3) (C) (1, 1) (D) (1, 5)
15. Equation of Tangent to  $x^2 + y^2 = 4$  at (2, 0) is: (A)  $x = 1$  (B)  $y = 1$  (C)  $y = 2$  (D)  $x = 2$
16. Slope of tangent to parabola  $y^2 = 4ax$  at (a, 2a) is: (A) 2 (B) -1 (C) 1 (D) 3
17. Eccentricity  $e$  of a circle is: (A)  $e = 0$  (B)  $e = 1$  (C)  $0 < e < 1$  (D)  $e > 1$
18. Radius of a circle  $x^2 + y^2 = 2$  is: (A) 2 (B) 1 (C)  $\frac{1}{2}$  (D)  $\sqrt{2}$
19. If  $P = (2, 3)$ ,  $Q = (6, -2)$ , then  $|\overline{PQ}|$  is: (A)  $\sqrt{40}$  (B)  $\sqrt{42}$  (C)  $\sqrt{41}$  (D)  $\sqrt{43}$
20. For a vector  $\vec{v} = 2\hat{i} + 3\hat{j} - 6\hat{k}$ , then  $\cos \beta = ?$  (A)  $\frac{3}{7}$  (B)  $\frac{2}{7}$  (C)  $-\frac{6}{7}$  (D)  $-\frac{3}{7}$

Mathematics (Subjective)

(For All Sessions)  
(GROUP-I)

Time: 2:30 hours

SECTION-I

Rwp-12-1-23

2. Write short answers of any eight parts from the following: (8x2=16)

- Express perimeter  $P$  of a square as a function of its area  $A$ .
- Evaluate  $\lim_{x \rightarrow 0} \frac{\sin x^0}{x}$
- Define even function with example.
- Find derivative by definition  $\frac{1}{\sqrt{x}}$
- If  $y = x^4 + 2x^2 + 2$ , prove that  $\frac{dy}{dx} = 4x\sqrt{y-1}$
- Differentiate w.r.t  $x$ ,  $y = \cot^{-1}\left(\frac{x}{a}\right)$
- Find  $\frac{dy}{dx}$ ;  $xy + y^2 = 2$
- Differentiate w.r.t  $x$ ,  $y = x\sqrt{\ln x}$
- Apply the Maclaurin series to prove that:  $e^{2x} = 1 + 2x + \frac{4x^2}{2!} + \frac{8x^3}{3!} + \dots$
- Define feasible region.
- Graph the solution set of  $2x + y \leq 6$ .

3. Write short answers of any eight parts from the following: (8x2=16)

- Evaluate  $\int \tan^2 x dx$ .
- Evaluate  $\int \frac{(a-b)x}{(x-a)(x-b)} dx$
- Evaluate  $\int x \sin x dx$ .
- Evaluate  $\int_{\pi/6}^{\pi/3} \cos t dt$
- Solve the differential equation  $y dx + x dy = 0$
- Evaluate  $\int \frac{x^2}{4+x^2} dx$
- Find the areas between the  $x$ -axis and the curve  $y = x^2 + 1$  from  $x = 1$  to  $x = 2$
- Find direction cosines of  $\underline{V} = 4\underline{i} - 5\underline{j}$
- Find a unit vector in the direction of  $\underline{V} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$
- Find  $\alpha$ , so that vector  $\underline{u} = 2\alpha\underline{i} + \underline{j} - \underline{k}$ ,  $\underline{v} = \underline{i} + \alpha\underline{j} + 4\underline{k}$  are perpendicular.
- Find the area of parallelogram whose vertices are:  $A(0, 0, 0)$   $B(1, 2, 3)$   $C(2, -1, 1)$   $D(3, 1, 4)$
- A force  $\underline{F} = 7\underline{i} + 4\underline{j} - 3\underline{k}$  is applied at  $p(1, -2, 3)$ . Find its amount about the point  $Q(2, 1, 1)$

4. Write short answers of any nine parts from the following: (9x2=18)

- Is  $(\sqrt{176}, 7)$  at a distance of 15 units from the origin?
- By means of slopes, show that the point  $(-4, 6)$ ,  $(3, 8)$ ,  $(10, 10)$  lie on the same line.
- Find  $K$  so that the line joining  $A(7, 3)$ ;  $B(k, -6)$  and the line joining  $C(-4, 5)$ ,  $D(-6, 4)$  are parallel.
- Find the equation of the line having  $y$ -intercept  $-7$  and slope  $-5$ .
- Find the point of intersection of the lines  $x - 2y + 1 = 0$  and  $2x - y + 2 = 0$
- Find equation of lines represented by  $2x^2 + 3xy - 5y^2 = 0$
- Find the measure of the angle between the lines represented by  $9x^2 + 24xy + 16y^2 = 0$
- Find an equation of the circle with ends of diameter at  $(-3, 2)$  and  $(5, -6)$
- Show that the line  $2x + 3y - 13 = 0$  is tangent to the circle  $x^2 + y^2 + 6x - 4y = 0$
- Check the position of the point  $(5, 6)$  with respect to the circle  $x^2 + y^2 = 81$ .
- Find focus and directrix of the parabola  $x^2 = -16y$
- Find an equation of ellipse if foci  $(-3\sqrt{3}, 0)$  and vertices  $(\pm 6, 0)$ .
- Find equation of hyperbola with given data foci  $(0, \pm 9)$ , directrices  $y = \pm 4$

SECTION-II

Note Attempt any three questions. Each question carries equal marks: (10x3=30)

- (a) Evaluate:  $\lim_{x \rightarrow 0} \frac{\sec x - \cos x}{x}$  (b) If  $y = \tan\left(2 \tan^{-1} \frac{x}{2}\right)$ , then show that  $\frac{dy}{dx} = 4\left(\frac{1+y^2}{4+x^2}\right)$
- (a) Evaluate:  $\int \frac{dx}{\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x}$  (b) Find equation of line through intersection of  $x + 2y + 3 = 0$ ,  $3x + 4y + 7 = 0$  and making equal intercepts on the axes.
- (a) Find the area bounded by the curve  $f(x) = x^3 - 2x^2 + 1$  and  $x$ -axis in the 1<sup>st</sup> quadrant.  
(b) Minimize  $Z = 3x + y$  subject to the constraints  $3x + 5y \geq 15$   $x + 6y \geq 9$   $x \geq 0$ ,  $y \geq 0$
- (a) If  $y = a \cos(\ln x) + b \sin(\ln x)$  prove that  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$   
(b) Find the coordinates of the points of intersection of the line  $2x + y = 5$  and the circle  $x^2 + y^2 + 2x - 9 = 0$ , also find the length of intercepted chord.
- (a) Find the centre foci, eccentricity and vertices of the ellipse  $x^2 + 16x + 4y^2 - 16y + 76 = 0$



Roll No. \_\_\_\_\_ to be filled in by the candidate

HSSC-(P-II)-A/2023

Paper Code 8 1 9 6

(For All Sessions)

(Group-II)

Time: 30 Minutes

Marks : 20

# Mathematics (Objective)

Rwp-12-2-23

Note: Write Answers to the Questions on the objective answer sheet provided. Four possible answers A, B, C and D to each question are given. Which answer you consider correct, fill the corresponding circle A, B, C or D given in front of each question with Marker or Pen ink on the answer sheet provided.

- 1.1 Midpoint of  $A(1,2)$  &  $B(3,8)$  is: (A)  $(2, 5)$  (B)  $(4, 10)$  (C)  $(2, 6)$  (D)  $(2, 8)$
2.  $(1, -3)$  is in the solution of \_\_\_\_\_ (A)  $x + y \geq 1$  (B)  $x + y \leq 0$  (C)  $x + y = 0$  (D)  $x - y = 0$
3. Centre of circle  $x^2 + y^2 - 6x + 4y + 13 = 0$  (A)  $(3, 2)$  (B)  $(-3, 2)$  (C)  $(3, -2)$  (D)  $(-3, -2)$
4. Focus of parabola  $x^2 = 4ay$  is: (A)  $(-a, 0)$  (B)  $(0, -a)$  (C)  $(a, 0)$  (D)  $(0, a)$
5. Eccentricity  $e$  for hyperbola is: (A)  $e = 1$  (B)  $e = 0$  (C)  $e < 1$  (D)  $e > 0$
6. Length of major axis of  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  (A) 03 (B) 06 (C) 02 (D) 04
7. Which one is not scalar quantity: (A) Work (B) Time (C) Magnetic field (D) Speed
8.  $[k \ i \ j]$  (A) 2 (B) 0 (C) 1 (D) -1
9.  $\lim_{x \rightarrow 2} \sqrt{x^3 + 1} - \sqrt{x^2 + 5}$  (A) -1 (B) 0 (C) 2 (D) -2
10. Area of circle of unit radius is: (A)  $\pi$  (B)  $2\pi$  (C)  $\pi^2$  (D)  $2\pi^2$
11.  $\frac{d}{dx}(3^x) =$  \_\_\_\_\_ (A)  $3^x \ln x$  (B)  $3^x \ln 2$  (C)  $3^x \ln 3$  (D)  $x 3^{x-1}$
12. Lagrange used \_\_\_\_\_ notation for derivative. (A)  $D f(x)$  (B)  $f^1(x)$  (C)  $\frac{d}{dx} f(x)$  (D)  $f'(x)$
13.  $\frac{d}{dx} \cos 7x =$  \_\_\_\_\_ (A)  $7 \sin 7x$  (B)  $-7 \sin 7x$  (C)  $7 \cos 7x$  (D)  $-7 \cos 7x$
14. Minimum value of function  $f(x) = x^2 + 2x - 3$  is at  $x =$  \_\_\_\_\_ (A) -3 (B) -2 (C) 0 (D) -1
15.  $\int \frac{1}{1+x^2} dx =$  \_\_\_\_\_ (A)  $\sin^{-1} x + c$  (B)  $\cos^{-1} x + c$  (C)  $\tan^{-1} x + c$  (D)  $\cot^{-1} x + c$
16.  $\int \frac{1}{x^2} dx =$  \_\_\_\_\_ (A)  $-\frac{1}{x} + c$  (B)  $\frac{1}{x} + c$  (C)  $\frac{2}{x} + c$  (D)  $-\frac{2}{x} + c$
17. Solution of  $\frac{dy}{dx} = 1$  is \_\_\_\_\_ (A)  $y = x^2 + c$  (B)  $y = e^x + c$  (C)  $y = \ln x + c$  (D)  $y = x + c$
18.  $\int_0^1 3x^2 dx =$  \_\_\_\_\_ (A) 3 (B) 1 (C) 2 (D) 0
19. Equation of line through origin with slope 2: (A)  $2x - y = 0$  (B)  $2x + y = 0$  (C)  $x + 2y = 0$  (D)  $x - 2y = 0$
20. Slope of line parallel to y-axis: (A) -1 (B) 0 (C)  $\infty$  (D) 1



**ematics (Subjective)**

(For All Sessions)

**(GROUP-II)**

Time: 2:30 hours

**SECTION-I**

*Rwp-12-2-23*

Write short answers of any eight parts from the following:

(8x2=16)

- i. Express perimeter P of a square as a function of its area A.
- ii. If  $f(x) = (-x + 9)^3$ , find  $f^{-1}(x)$
- iii. Find  $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$
- iv. Differentiate w.r.t "x"  $(\sqrt{x} - \frac{1}{\sqrt{x}})^2$
- v. If  $y = \sqrt{x + \sqrt{x}}$  find  $\frac{dy}{dx}$
- vi. Find  $\frac{dy}{dx}$  if  $x = y \text{ Siny}$
- vii. Find  $f'(x)$  if  $f(x) = x^3 \cdot e^{1/x}$
- viii. If  $y = x^2 \cdot \ln(\frac{1}{x})$ , find  $\frac{dy}{dx}$
- ix. If  $y = \text{Sin } h^{-1}(\frac{x}{2})$ , Find  $\frac{dy}{dx}$
- x. Apply the Maclaurin series to prove that:  $\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \dots$
- xi. Graph the solution set of linear inequality in  $xy$  - plane,  $2x + y \leq 6$
- xii. What is a feasible solution?

3. Write short answers of any eight parts from the following:

(8x2=16)

- i. Using differentials find  $\frac{dy}{dx}$  and  $\frac{dx}{dy}$  for  $x^2 + 2y^2 = 16$
- ii. Evaluate:  $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$
- iii. Evaluate:  $\int \frac{x+2}{\sqrt{x+3}} dx$
- iv. Evaluate:  $\int \tan^{-1} x dx$
- v. Evaluate:  $\int \frac{5x+8}{(x+3)(2x-1)} dx$
- vi. Evaluate:  $\int_2^0 \frac{1}{(2x-1)^2} dx$
- vii. Solve the differential equation  $\frac{dy}{dx} = \frac{y^2+1}{e^{-x}}$
- viii. Find sum of  $\overline{AB}$  and  $\overline{CD}$  where  $A(1, -1)$ ,  $B(2, 0)$ ,  $C(-1, 3)$  and  $D(-2, 2)$
- ix. Find direction Cosines of vector  $\underline{V} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$
- x. Find  $\alpha$  so that  $\underline{U} = 2\alpha \mathbf{i} + \mathbf{j} - \mathbf{k}$  and  $\underline{V} = \mathbf{i} + \alpha \mathbf{j} + 4\mathbf{k}$  and perpendicular.
- xi. Compute  $\underline{a} \times \underline{b}$  and  $\underline{b} \times \underline{a}$  for  $\underline{a} = \mathbf{i} + \mathbf{j}$ ,  $\underline{b} = \mathbf{i} - \mathbf{j}$
- xii. Find volume of parallelepiped determined by  $\underline{U} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ ,  $\underline{V} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$  and  $\underline{w} = \mathbf{i} - 7\mathbf{j} - 4\mathbf{k}$

4. Write short answers of any nine parts from the following:

(9x2=18)

- i. The point  $C(-5, 3)$  is the center of the circle and  $P(7, 2)$  lies on the circle. What is the radius of the circle.
- ii. Show that the points  $A(0, 2)$ ,  $B(\sqrt{3}, -1)$  and  $C(0, -2)$  are vertices of a right triangle.
- iii. The points  $P(-2, 6)$  and  $O(-3, 2)$  are given in  $xy$  - coordinate system. Find the  $XY$  - Coordinate of P referred to the translated axes  $OX'$  and  $OY'$ .
- iv. Find an equation of the line through  $(-5, -3)$  and  $(9, -1)$ .
- v. Convert  $4x + 7y - 2 = 0$  in slope-intercept form.
- vi. Find the lines represented by  $3x^2 + 7xy + 2y^2 = 0$
- vii. Find the point of intersection of the lines  $3x + y + 12 = 0$  and  $x + 2y - 1 = 0$
- viii. Find center and radius of circle  $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
- ix. Find focus and vertex of parabola  $y^2 = -12x$
- x. Find foci of an ellipse  $9x^2 + y^2 = 18$
- xi. Find eccentricity of hyperbola,  $\frac{y^2}{4} - x^2 = 1$
- xii. Write parametric equations of hyperbola.
- xiii. Write down equation of tangent to the circle  $x^2 + y^2 = 25$  at  $(4, 3)$ .

**SECTION-II**

(10x3=30)

Note Attempt any three questions. Each question carries equal marks:

- 5. (a) Evaluate:  $\lim_{x \rightarrow 0} \frac{\text{Sec } x - \text{Cos } x}{x}$
- (b) Find  $\frac{dy}{dx}$  if  $x\sqrt{1+y} + y\sqrt{1+x} = 0$ .
- 6. (a) Evaluate:  $\int \frac{x}{x^4 + 2x^2 + 5} dx$
- (b) Find equation of the line through  $(5, -8)$  and perpendicular to the join of  $A(-15, -8)$  and  $B(10, 7)$ .
- 7. (a) Solve the differential equation  $(y - x \frac{dy}{dx}) = 2(y^2 + \frac{dy}{dx})$
- (b) Graph the feasible region of the following system of linear inequalities and find the corner points.  
 $2x + y \leq 10, x + 4y \leq 12, x + 2y \leq 10, x \geq 0, y \geq 0$
- 8. (a) Show that  $y = \frac{\ln x}{x}$  has maximum value at  $x = e$ .
- (b) Write an equation of the circle that passes through the given points  $A(4, 5)$ ,  $B(-4, -3)$ ,  $C(8, -3)$
- 9. (a) Find the focus, vertex and directrix of the parabola  $x^2 - 4x - 8y + 4 = 0$